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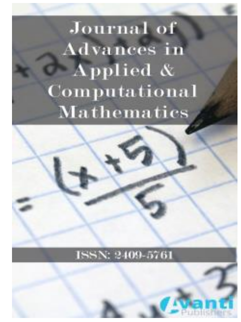

Fuzzy Rough Subgroups on Approximation Space

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ABSTRACT

Fuzzy rough sets are a mathematical concept that combines fuzzy sets and rough sets to deal with uncertainty and incompleteness in data and information. In this study, different from the definition of Dubois and Prade (1990), the fuzzy rough set is defined within the framework of the rough group concept defined by Biswas and Nanda (1994), and some of its algebraic properties are discussed. Then, the concepts of fuzzy rough subgroup and fuzzy rough normal subgroup are introduced in the rough group. In addition, some basic features and examples of these concepts are given.

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1. Introduction

To deal with vagueness, rough sets, and fuzzy sets are two efficient set theories. Both are generalizations of classical sets but have different viewpoints and applications.

Fuzzy sets are first introduced by Lotfi Zadeh [1], allowing objects to belong to a set or relation to a given degree, this is called the degree of membership. Fuzzy sets have been applied to various domains, such as logic, control, decision-making, and artificial intelligence. Fuzzy sets allow us to represent linguistic terms such as "a lot", "more or less" or "about" with a numerical value. Fuzzy sets have been applied to various mathematical fields. For example, fuzzy subgroups were defined and established by Rosenfeld [2]. Then many authors have studied it [3, 4].

Rough sets introduced by Polish computer scientist Z. Pawlak in [5], provide approximations of concepts in the presence of missing information. Although it is a generalization of classical sets, it uses a pair of sets to approximate the original set. The lower approximation includes objects that belong to the set, while the upper approximation includes objects that probably belong to the set. At the same time, some researchers have applied this theory to algebraic structures as well. Some algebraic properties of rough sets were explored by Iwinski [6]. In [7], Kuroki and Wang introduced the upper and lower approximations together with normal subgroups in a group. Then, some features of upper and lower approximations were studied according to normal subgroups [8-12]. On the other hand, in [13] definitions of the notation of rough subgroups and rough groups are given by using only the upper approximation. Miao *et al*. developed the rough group and rough subgroup definitions and offered some new characteristics [14]. In [15], the notation of rough semigroup is introduced. Also, Bağırmaz *et al*. defined the concept of the topological rough group [16]. Li *et al*, separation axioms of topological rough groups are discussed in [17].

On the other hand, some researchers have tried to reduce the limitations of equivalence relations in Pawlak rough sets in practice. Therefore, many researchers have proposed some general models [18-21]. Later, combining rough sets with fuzzy sets, Dubois and Prade [21] introduced the concepts of rough fuzzy sets and rough fuzzy sets. From this point of view, some researchers have applied this idea to other areas of mathematics [22-24].

This study is regulated as follows. In section 2, basic notations of fuzzy subgroups and rough groups are given. In section 3, fuzzy rough subset and (normal) subgroup definitions were made and some important features were proved.

2. Preliminaries

This section is dedicated to present some definitions and propositions that will form the basis for subsequent chapters.

Definition 2.1. [5] Assume U be a non-empty finite set called universe and R be an equivalence relation on U. Then (U, R) is called an approximation space.

Definition 2.2. [5] Assume U be a universe and R be an equivalence relation on U. We denote the equivalence class of object x in R by $[x]_R$.

Definition 2.3. [5] Assume (U, R) be an approximation space and X be a subset of U. The sets

(i) $\overline{X} = \{x | [x]_R \cap X \neq \emptyset\}$,

(ii) $\underline{X} = \{x | [x]_R \subseteq X\}$,

are called upper approximation and lower approximation of X in (U,R), respectively.

For example, suppose that (U,R) is an approximation space, where $U = \{a, b, c, d, e, f\}$ and an equivalence relation R with the following equivalence classes:

$$C_1 = \{a, b\}, \quad C_2 = \{c, e, f\}, \qquad C_3 = \{d\}.$$

Let $A = \{a, b, e\}$. Then $\underline{A} = \{a, b\}$ and $\overline{A} = \{a, b, c, e, f\}$.

Let U be a universe and "*" a binary operation defined on U. From now on, ab will be used instead of a*b, for all $a, b \in U$, and U denote a universe set in (U,R).

Definition 2.4. [13] Assume (U,R) be an approximation space and $G \subseteq U$. Then, G is called a rough group if the following properties are satisfied:

(i) $\forall a, b \in G, ab \in \overline{G}$,

(ii) Associativity property holds in $\overline{G}$

(iii) $\exists e \in \overline{G}$, $\forall a \in G$ such that $ae = ea = a$, where e is called the rough identity element of rough group G.

(iv) $\forall a \in G, \exists b \in G$ such that $ab = ba = e$, where b is called the rough inverse element of a in G, it is denoted by a^{-1}.

Definition 2.5. [13] Assume G be a rough group and $H \subseteq G$. H is called a rough subgroup of G if H is a rough group.

Remark 2.6. [13] Assume G be a rough group. Then, G has only one rough subgroup, which is itself. A necessary and sufficient condition for {e} to be a trivial rough subgroup of G is $e \in G$.

Proposition 2.7. [13] Assume G be a rough group and $H \subseteq G$. A necessary and sufficient condition for H to be a rough subgroup is that:

(i) $\forall a, b \in H, ab \in \overline{H}$,

(ii) $\forall a \in H, a^{-1} \in H$.

Definition 2.8. [1] Assume U be a non-empty set. A fuzzy subset φ of U is a map $\phi: U \to [0,1]$.

Definition 2.9. [3] A fuzzy set φ of a group G is called a fuzzy subgroup if, for all $a, b \in G$,

(i) $\phi(ab) \geq \min\{\phi(a), \phi(b)\}$,

(ii) $\phi(a^{-1}) \geq \phi(a)$.

It is well known that a fuzzy subgroup G satisfies $\phi(a) \leq \phi(e)$ and $\phi(a^{-1}) = \phi(a)$ for all $a \in G$.

3. Fuzzy rough subgroups

In this part, a fuzzy rough (sub) set and a fuzzy rough (normal) subgroup of rough groups are defined. In addition, some important properties were proved and an example was given.

Definition 3.1. Assume G be a non-empty subset of U. A fuzzy rough set φ of $\overline{G}$ is a map $\varphi : \overline{G} \to [0,1]$.

Definition 3.2. Assume G be a non-empty subset of U. Let $\varphi : \overline{G} \to [0,1]$ is a map defined as

$$\phi(a) = \begin{cases} g_1, & a \in G, \\ g_2, & a \in \overline{G} \setminus G, 0 \leq g_1 \leq g_2 \leq 1 \end{cases}$$

where $g_1, g_2 \in [0,1]$. The set φ is known as fuzzy rough subset of $\overline{G}$.

Definition 3.3. Assume G be rough group over U. A fuzzy rough subset φ of $\overline{G}$ is named a fuzzy rough subgroup of G if, for all $a, b \in G$,

(i) $\phi(ab) \geq \min\{\phi(a), \phi(b)\}$,

(ii) $\phi(a^{-1}) \geq \phi(a)$.

Proposition 3.4. Assume G be a rough group over U. If ϕ is a fuzzy rough subgroup of G, then:

(i) $\phi(e) \geq \phi(a), \forall\, a \in G$,

(ii) $\phi(a^{-1}) = \phi(a), \forall\, a \in G$,

where e is the identity element of G.

Proof. (i) Suppose that φ be a fuzzy rough subgroup of G, then $\phi(e) = \mu(aa^{-1}) \geq \min\{\phi(a), \phi(a^{-1})\} = \phi(a)$ for all $a \in G$. Thus $\phi(e) \geq \phi(a)$.

(ii) Suppose that φ be a fuzzy rough subgroup of G, then $\phi(a) = \phi((a^{-1})^{-1}) \geq \phi(a^{-1}) \geq \phi(a)$ for all $a \in G$. Hence $\phi(a^{-1}) = \phi(a)$.

Definition 3.5. Assume G be a rough group over U. A fuzzy rough subgroup ϕ of G is named a fuzzy rough normal subgroup of G if

$\phi(ab) = \phi(ba)$ for all $a, b \in G$.

Example 3.6. Suppose that $U = \{a, b, c, d, e\}$ be a universe set with the following multiplication table:

(·)	a	b	c	d	e
a	a	b	c	d	e
b	b	a	c	b	b
c	c	c	c	a	e
d	d	c	a	d	c
e	e	d	b	c	e

A classification of U is $U/R = \{C_1, C_2, C_3\}$, where

$$C_1 = \{a, b\},$$

$$C_2 = \{c, d\},$$

$$C_3 = \{e\}.$$

Let $G = \{b, c, d\}$, then $\overline{G} = \{a, b, c, d\}$. From Definition 2.4 , $G \subseteq U$ is a rough group.

For $\overline{G}$ define $\phi(a) = 1$, $\phi(b) = \frac{1}{2}$, $\phi(c) = \phi(d) = \frac{1}{3}$. Then, from Definition 3.3, ϕ is a fuzzy rough subgroup of G.

Proposition 3.7. Assume G be a group over U. If φ is a fuzzy (normal) subgroup of G, then ϕ is a fuzzy rough (normal) subgroup of G.

Proof. Suppose that G be a group. Then $a, b \in G$, $ab \in G$. Since $G \subseteq \overline{G}, ab \in \overline{G}$. On the other hand, since ϕ is a fuzzy subgroup of G, then from Definition 3.3 we get ϕ is a fuzzy rough subgroup of G.

Similarly, since ϕ is a fuzzy normal subgroup of G, then from Definition 3.5 we conclude ϕ is a fuzzy rough normal subgroup of G.

Remark 3.8. Assume G be a group over U. Obviously, G is also a rough group over U.

Lemma 3.9. Assume G be a rough group over U and $\mathrm{G}=\overline{\mathrm{G}}$. Then G is a group.

Proof. This is easily obtained from Proposition 19 [16].

Proposition 3.10. Assume G be a rough group over U and $\mathrm{G}=\overline{\mathrm{G}}$. If ϕ is a fuzzy rough subgroup of G, then ϕ is a fuzzy subgroup of G.

Proof. Suppose that G be a rough group and $\mathrm{G}=\overline{\mathrm{G}}$. Then, from Lemma 3.9 we get G is a group. Thus $\mathrm{a},\mathrm{b}\in \mathrm{G}$, $\mathrm{ab}\in\mathrm{G}$, and so $\phi\colon \mathrm{G}\to [0,1]$ is a fuzzy subset of G. Since ϕ is a fuzzy rough subgroup of G, then from Definition 3.3 we conclude ϕ is a fuzzy subgroup of G.

4. Conclusion

In this study, in the context of group theory a bridge has been established between fuzzy sets and rough sets. Fuzzy rough (sub) set, fuzzy rough (normal) subgroups have introduced. In addition, important and basic properties of these concepts were examined.

Conflict of Interest

The author declares no conflict of interest.

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